Primordial black holes and induced gravitational waves in (string) inflation

Ivonne Zavala

Swansea University

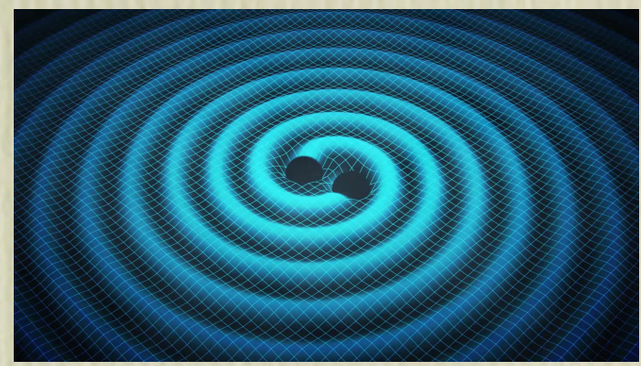
String Pheno 2022

Liverpool, UK, July 2022

Aragam, Bhattacharya, Chakraborty, Chiovoloni, Loaiza, Niz, Ozsoy, Paban, Parameswaran, Rosati, Tasinato

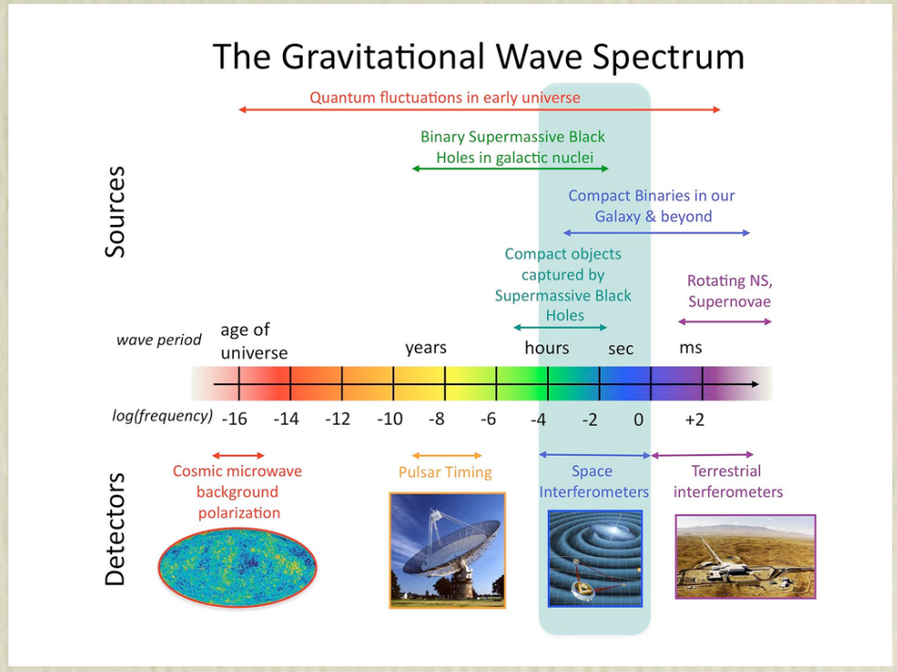
GRAVITATIONAL WAVE COSMOLOGY

- The recent first detection of ***gravitational waves*** (GW) in the fabric of spacetime from ***black hole*** binaries, has opened up a new way to study our Universe and new avenues to study cosmology.



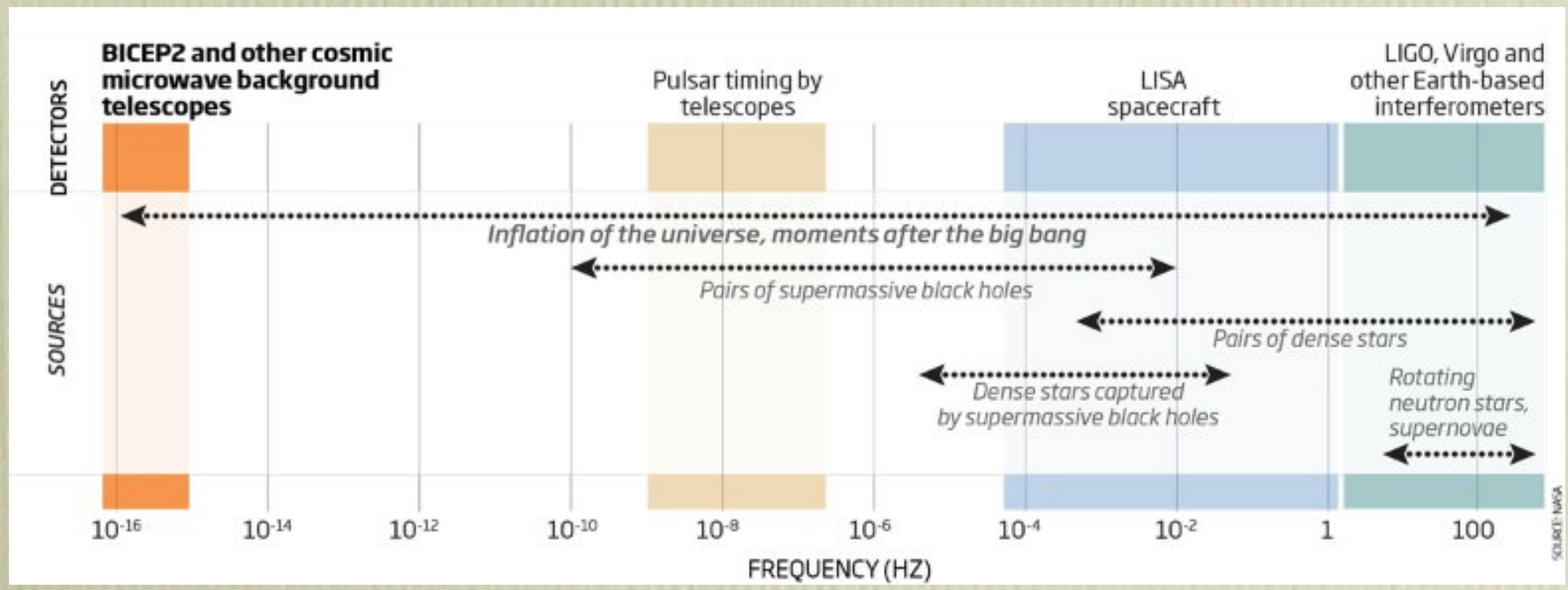
[LIGO collaboration, '15]

- One very exciting, but challenging prospect, is the measurement of ***primordial gravitational waves*** (PGW) produced in the very early universe during ***cosmological inflation*** $(t \sim 10^{-34}\mathrm{sec})$



PRIMORDIAL GRAVITATIONAL WAVES

- PGWs are a generic ***prediction of cosmological inflation***. Amplitude is model dependent
- Cosmic Microwave Background (CMB) polarisation experiments can probe their amplitude at very ***large*** CMB ***scales***.
- Smaller scales not constrained by CMB



PRIMORDIAL BLACK HOLES

(see Anguelova, Cicolli, Lüst, Mavromato's talks)

- Large primordial fluctuations may collapse to form ***primordial black holes*** (PBHs).

[Zeldovich, Novikov, '66]
[García-Bellido, Linde, Wands, '96]

- ***PBHs*** can provide a significant fraction – or all – of the mysterious ***Dark Matter*** has become a serious possibility.

[See e.g. Carr, Kuhnel, '20; Green, Kavanagh, '20]

- The ***amplitude*** of inflationary perturbations at the scales probed by the ***CMB*** is ***constrained*** by observations to be

$$A_s \sim 10^{-9} \quad @ \quad k = 0.05\,\mathrm{Mpc}^{-1}$$

- For inflationary perturbations to produce a significant PBH population, it needs a ***mechanism to enhance*** them to $A_s \sim 10^{-2}$ at smaller scales.

[Garcia-Bellido, Ruiz-Morales; Ezquiaga, Garcia-Bellido, Ruiz-Morales; Ballesteros, Taoso; Hertzberg,Yamada; Kannike, L. Marzola, M. Raidal and H. Veermäe, '17]

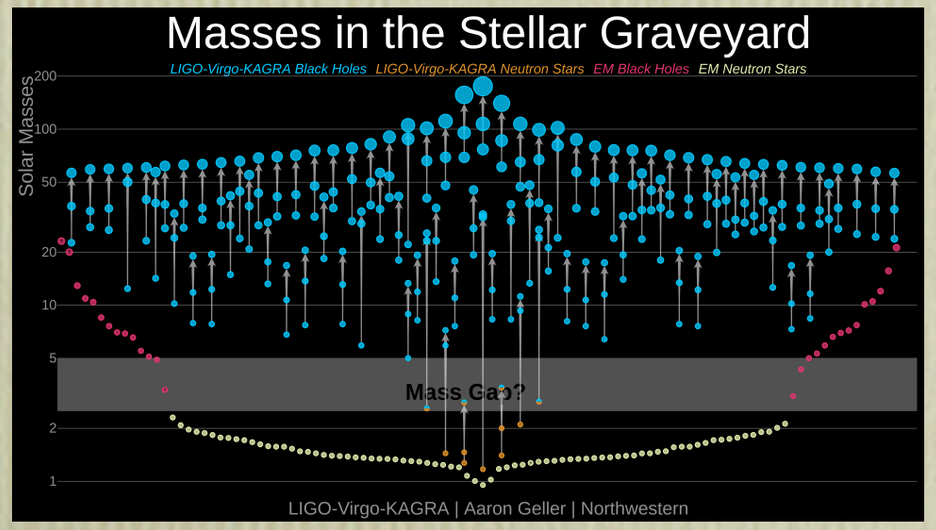
PRIMORDIAL BLACK HOLES AND IGWS

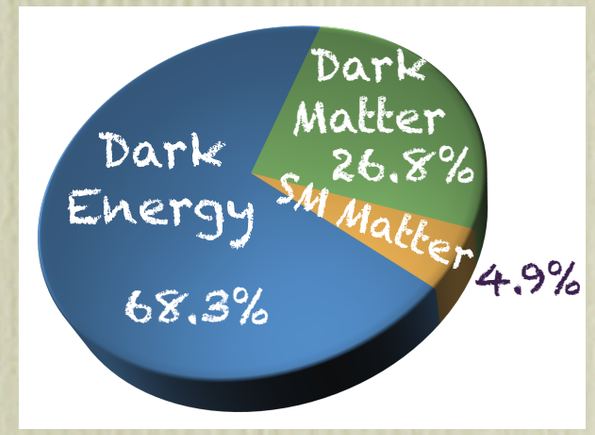
(Anguelova, Cicolli, Lüst, Muia, Mavromato's talks)

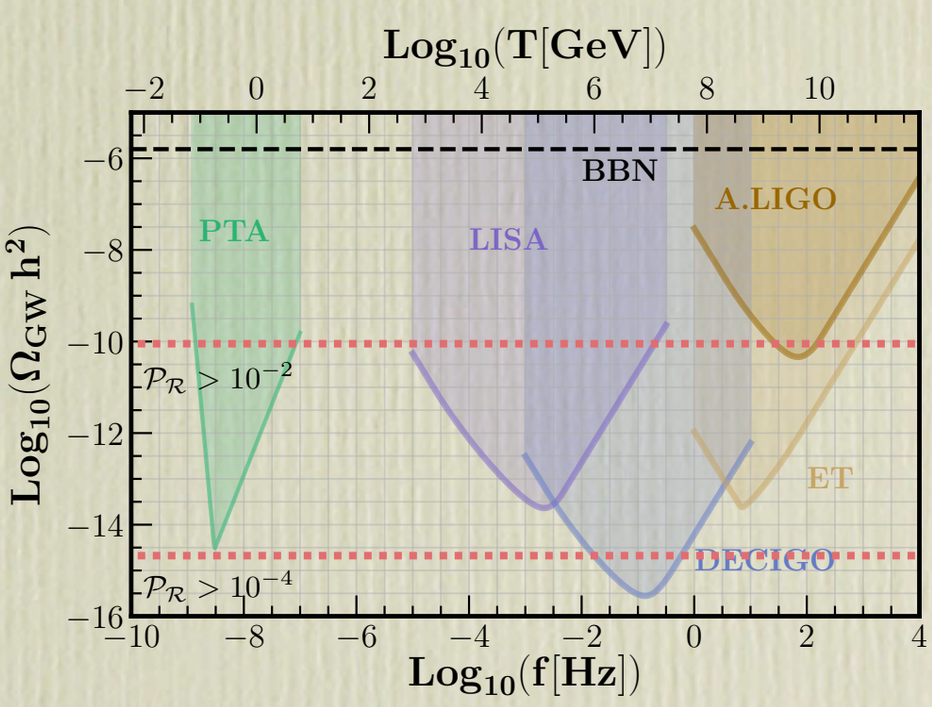
At first order scalar and tensor fluctuations are decoupled. At second order, ***GWs*** are ***induced*** from first-order scalar perturbations. If large, potentially detectable at interferometers

[Tomita, '67]
[review: Domenech, '21]

GWs induced by primordial fluctuations, and PBHs, are a direct window to the latest stages of inflation.







[Thrane, Romano, '13; Domenech, '21]

PLAN

- Amplification of the curvature perturbations:
  - single field inflation, PBHs and IGWs
- Beyond single field:
  - large turn attractor
  - sharp turns, PBHs and IGWs
- Summary

ENHANCEMENT OF THE CURVATURE PERTURBATION

Consider the mode equation of curvature perturbation:

[Garriga-Mukhanov, '99]

$$\mathcal{R}_k'' + 2\frac{z'}{z}\mathcal{R}_k' + c_s^2\, k^2 \mathcal{R}_k = 0,$$

$$z = \frac{a\dot{\phi}}{Hc_s}\,, \quad \frac{z'}{z} = aH\left(1+\epsilon-\delta-s\right)\,, \quad \epsilon = -\frac{\dot{H}}{H^2}\,, \qquad \delta = -\frac{\ddot{H}}{2H\dot{H}}\,, \qquad s = \frac{\dot{c}_s}{Hc_s}\,.$$

Solution for modes that already left the horizon $(k < |z'/z|)$ is given by

$$\mathcal{R}_k(\tau) = \mathcal{C}_1 + \mathcal{C}_2 \int \frac{\mathrm{d}\tau}{z^2} \qquad (\mathcal{C}_1\,,\ \mathcal{C}_2 = \mathrm{const.})$$

with

$$z(a) = z_0 \exp\left[\int (1+\epsilon-\delta-s)\, d\ln a\right]$$

ENHANCEMENT OF THE CURVATURE PERTURBATION

Solution for modes that already left the horizon

$$\mathcal{R}_k(\tau) = \mathcal{C}_1 + \mathcal{C}_2 \int \frac{\mathrm{d}\tau}{z^2}$$

In the **slow-roll** limit,

$$\epsilon,\ \delta,\ s \ll 1 \quad \Rightarrow \quad z \sim a \quad \Rightarrow \quad \mathcal{C}_2 \int \frac{\mathrm{d}\tau}{z^2}$$

decaying mode

ENHANCEMENT OF THE CURVATURE PERTURBATION

Solution for modes that already left the horizon

$$\mathcal{R}_k(\tau) = \mathcal{C}_1 + \mathcal{C}_2 \int \frac{\mathrm{d}\tau}{z^2}$$

In the **slow-roll** limit,

$$\epsilon,\ \delta,\ s \ll 1 \quad \Rightarrow \quad z \sim a \quad \Rightarrow \quad \mathcal{C}_2 \int \frac{\mathrm{d}\tau}{z^2}$$

decaying mode

Slow-roll violation to enhance power: $\frac{z'}{z} < 0\,,$

$$1 + \epsilon - \delta - s < 0$$

decaying mode ⇨ growing mode!

Enhancement of the curvature perturbation

[Leach, Liddle,'00;Leach, Sasaki, Wands, Liddle, '01]
[Özsoy, Parameswaran, Tasinato, IZ, '18]

ENHANCEMENT OF THE SCALAR PERTURBATION SPECTRUM IN STRING INFLATION

- String axion inflation including subleading non-perturbative corrections (single field)

ENHANCEMENT MECHANISMS IN STRING INFLATION

- Axion monodromy with subleading non-perturbative corrections

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \Lambda_1^4 \, \frac{\phi}{f} \, \cos\left(\frac{\phi}{f}\right) + \Lambda_2^4 \sin\left(\frac{\phi}{f}\right)$$

[Westphal, Silverstein, '08; Kobayashi, Oikawa, Otsuka, '15; Cabo-Bizet, Loaiza-Btito, IZ, '16;

$$\beta_i \equiv \Lambda_i^4/m^2 f^2 \qquad (i = 1,2)$$

- $\beta_i > 1$, large number of new stationary points ➟ inflation field might get stuck in some local minimum

[Banks-Dine-Fox-Gorbatov, '03]

- $\beta_i \ll 1$, tiny corrections $\Rightarrow$ inflaton's trajectory hardly affected but imprints in CMB

[Westphal-Silverstein-McAllister, '08; Kobayashi-Takahashi, '10; Kappl-Nilles-Winkler, '15; Choi-Kim, '15]

ENHANCEMENT MECHANISMS IN STRING INFLATION

- Axion monodromy with subleading non-perturbative corrections

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \Lambda_1^4 \, \frac{\phi}{f} \, \cos\left(\frac{\phi}{f}\right) + \Lambda_2^4 \sin\left(\frac{\phi}{f}\right)$$

[Westphal, Silverstein, '08; Kobayashi, Oikawa, Otsuka, '15; Cabo-Bizet, Loaiza-Btito, IZ, '16;

$$\beta_i \equiv \Lambda_i^4/m^2 f^2 \qquad (i = 1, 2)$$

- Interesting case arises for $\beta_i \lesssim 1$

[Parameswaran, Tasinato, IZ, '16; Özsoy, Parameswaran, Tasinato, IZ, '18]

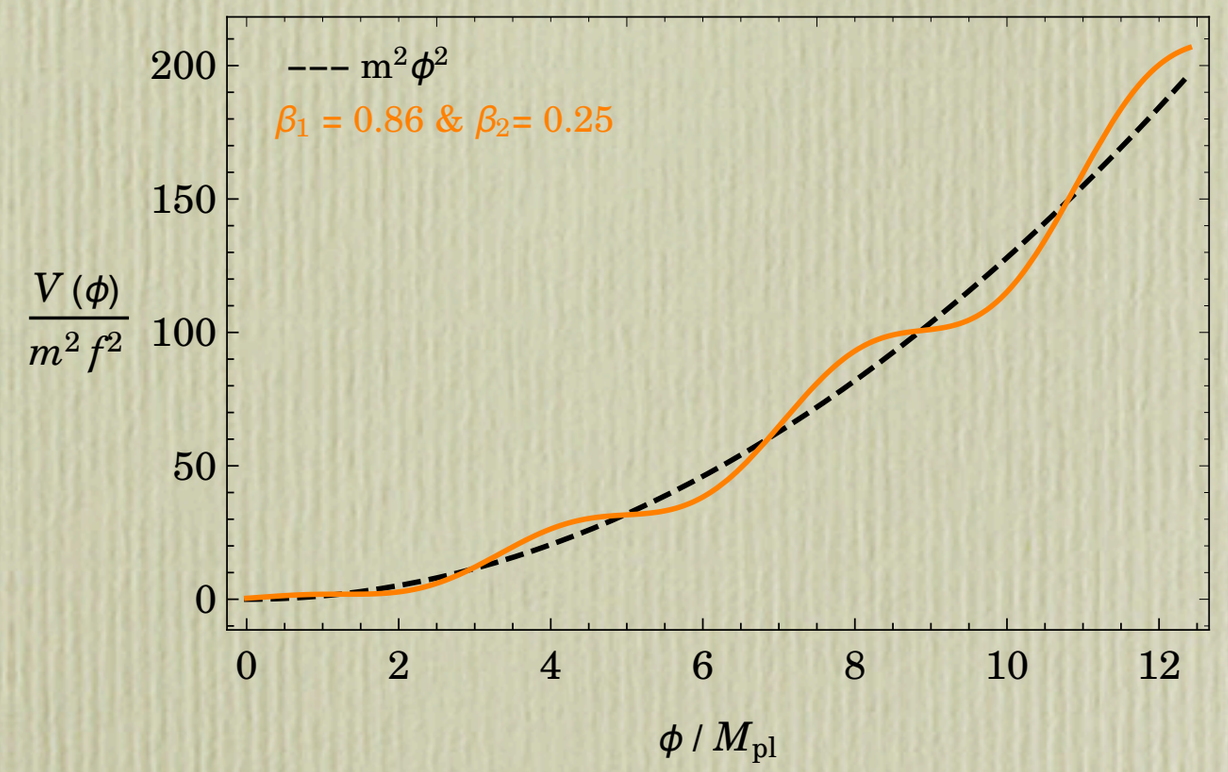
Subleading but non-negligible corrections. Steep cliffs and gentle plateaus with interesting phenomenology.

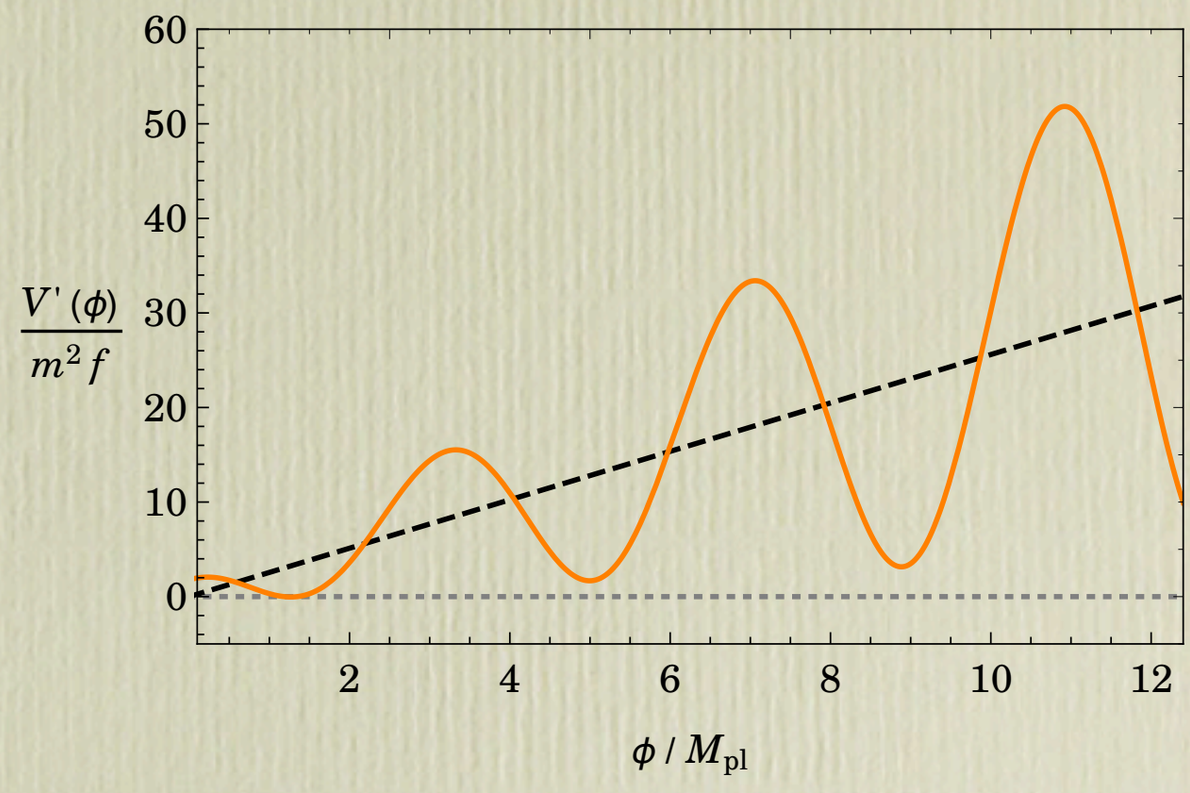
The cliffs can momentarily violate the *slow-roll* conditions, and the plateaus can lead to phases of *ultra-slow-roll* inflation: $\epsilon \ll 1\,,\ \delta \gtrsim 3\,,\ (V' \sim 0)$

Large enhancement of curvature perturbation!

PBHS IN AXION MONODROMY

Özsoy, Parameswaran, Tasinato, IZ, '18]





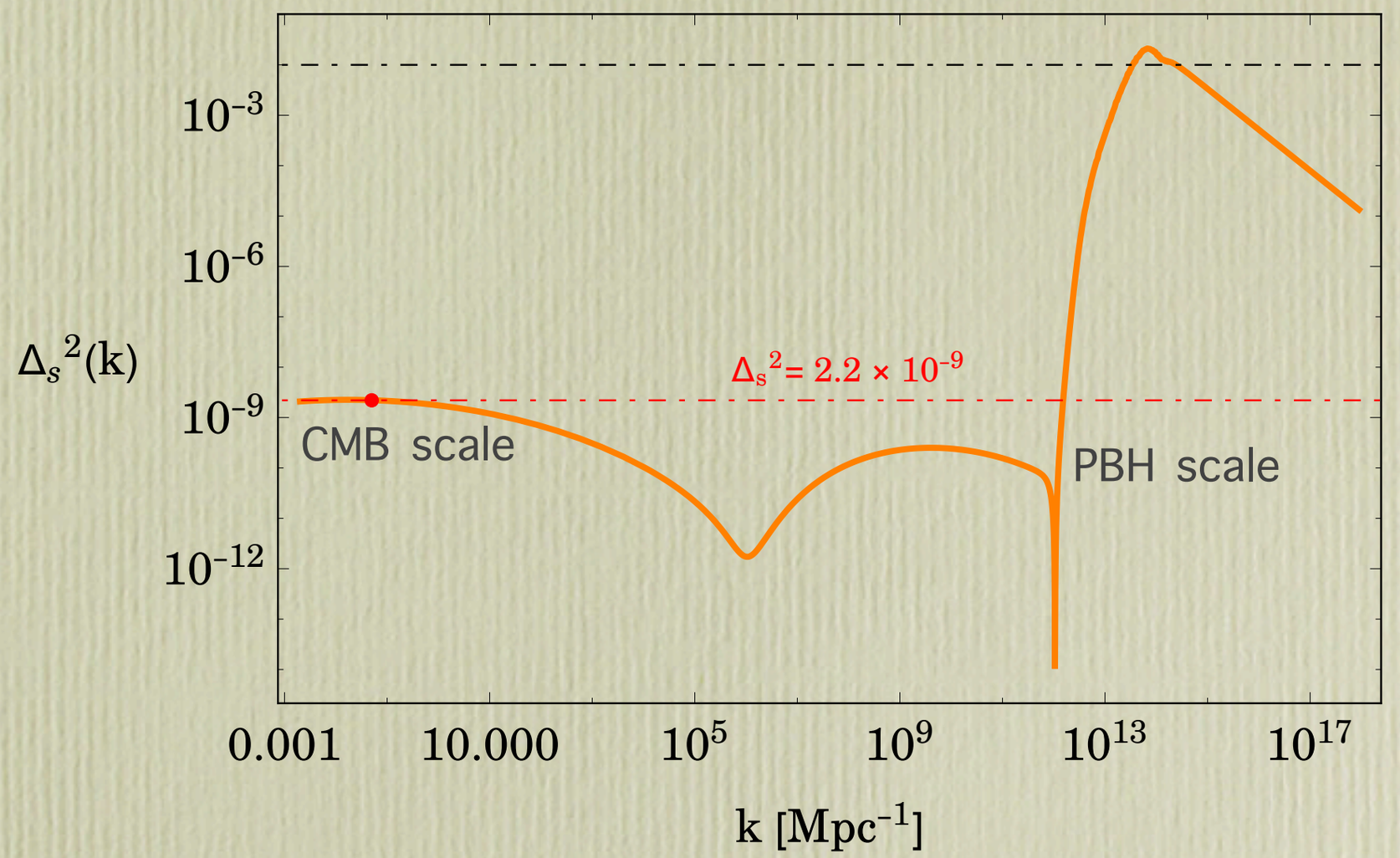
$$\beta_1 \simeq 0.86\,, \qquad \beta_2 \simeq 0.25\,, \qquad M_{pl}/f = 1.6$$

inflection point at small scales

PBHS IN AXION MONODROMY

Özsoy, Parameswaran, Tasinato, IZ, '18]

$\beta_1 \simeq 0.86\,, \qquad \beta_2 \simeq 0.25\,, \qquad M_{\rm pl}/f = 1.6$



Large enhancement at large scales

PBHs formed due to gravitational collapse of large fluctuations upon horizon re-entry during *radiation epoch*

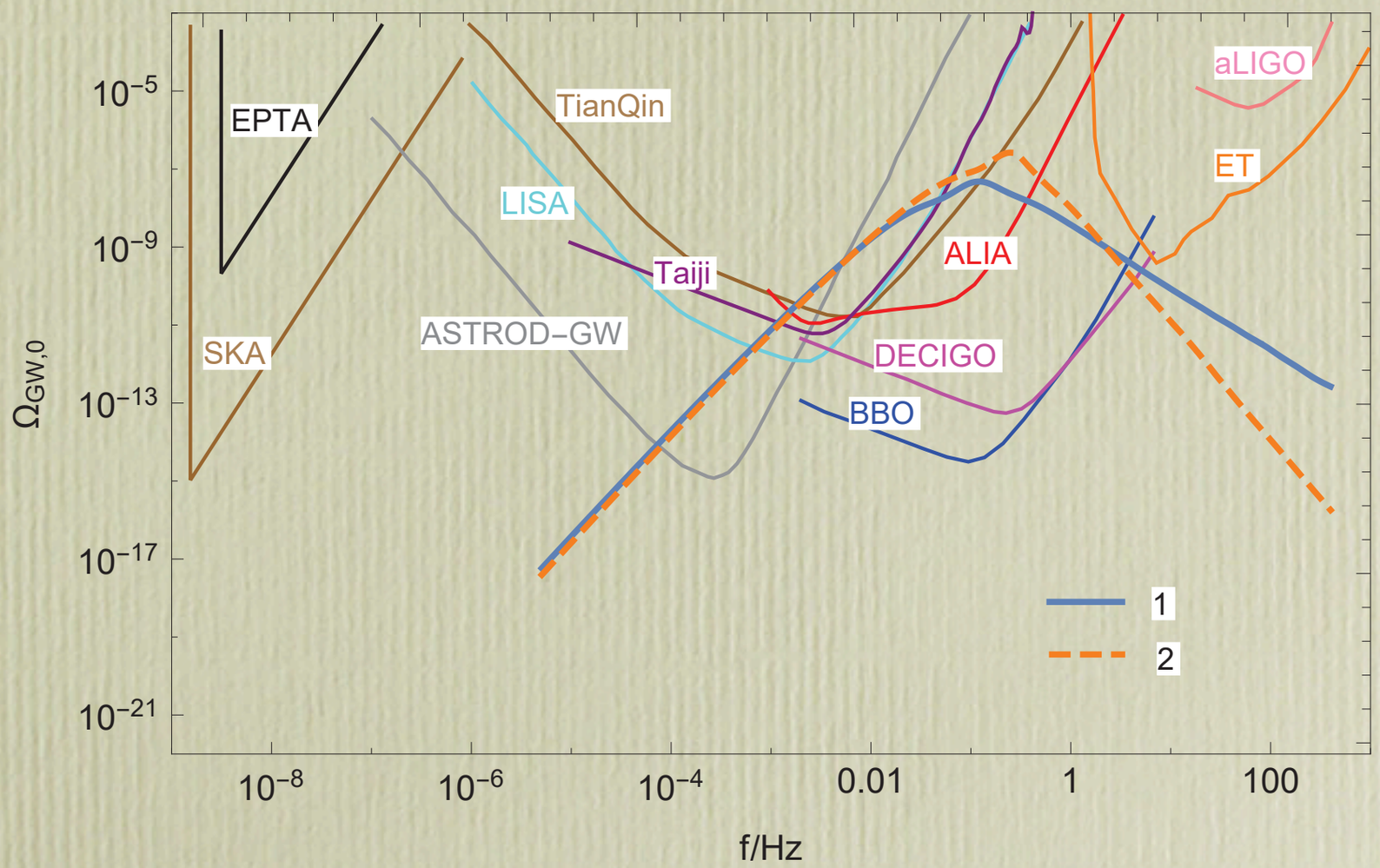
Cases	δ_c	$M_{\rm peak}/M_\odot$	$\Omega^{\rm tot}_{\rm PBH}/\Omega_{\rm DM}$
$M_{\rm pl}/f = 1.6$	0.34	8×10^{-16}	0.113
$M_{\rm pl}/f = 1.7$	0.5	2×10^{-16}	0.514

IGWS IN AXION MONODROMY

[Gao, Yang, '19]

Large scalar perturbation can *source* GWs at *second order*

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = S_{\mathbf{k}}(\eta) \qquad (S_{\mathrm{k}} \sim \mathcal{R}_{\mathrm{k}}^2)$$



$$\Omega_{GW}^{\mathrm{ind}} \sim 10^{-6}\mathcal{P}_{\mathcal{R}}^2$$

$$(k \gg k_{\mathrm{CMB}})$$

Case	M_{pl}/f	$V_0^{1/4}$	m	Λ_1	Λ_2
1	1.6	9.98285×10^{-4}	$2.563395477 \times 10^{-6}$	1.218164×10^{-3}	8.9630812×10^{-4}
2	1.7	8.64799×10^{-4}	$2.12915358 \times 10^{-6}$	$1.08856568 \times 10^{-3}$	7.1318712×10^{-4}

BEYOND SINGLE FIELD

(Anguelova's talk)

Features via deviation from slow-roll require non-trivial fine-tuned inflection points/bumps in potential.

Inflation is likely to be described by BSM theories, supergravity and string theory. Usually there are multiple degrees of freedom that could be relevant for inflation

Effective single field behaviour at large scales (near CMB pivot) can satisfy Planck constraints.

Features and deviations from slow-roll can be generated by inflection points/bumps induced by additional field(s) at smaller scales AND non-trivial field space dynamics.

MULTI FIELD SLOW-ROLL INFLATION

(Anguelova's talk)

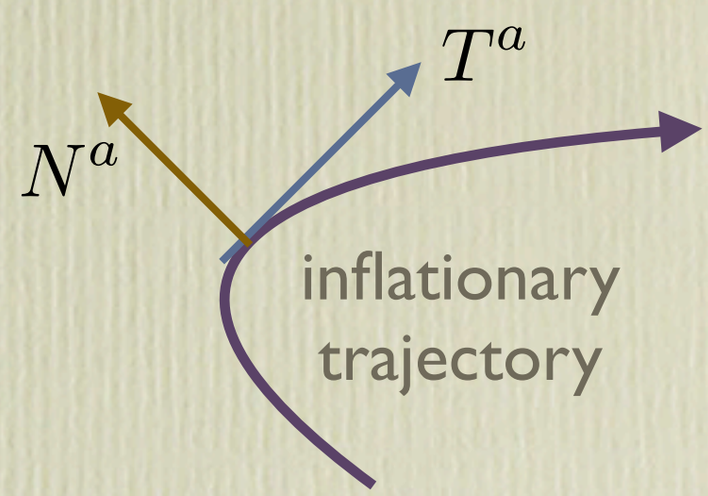
New inflationary attractor arises:

[Achúcarro, Bjorkmo, Brown, Hetz, Palma, Christodoulidis, Marsh, Roest, Renaux-Petel , Sfakianakis, Turzyński, 15-19]

- **Large slow turning** (strongly non-geodesic) inflation

$$\omega \equiv \frac{\Omega}{H} \quad = \quad \text{Turning rate}$$

$$\dot{\varphi}^2 = g_{ab}\dot{\phi}^a\dot{\phi}^b$$



- New slow-roll parameter: $\nu \equiv \frac{\dot{\omega}}{H\omega} \qquad (\nu \ll 1)$

[Aragam, Paban, Rosati, '20]

- Inflatons' masses can be large; steep potential ok

$$\eta_V \equiv M_{\rm Pl}^2 \left| \text{min eigenvalue} \left(\frac{\nabla^a \nabla_b V}{V} \right) \right| \gg 1$$

[Chakraborty et al. '19;
Aragam, Chivoloni, Paban, Rosati, IZ, '21]

MULTI FIELD SLOW-ROLL INFLATION

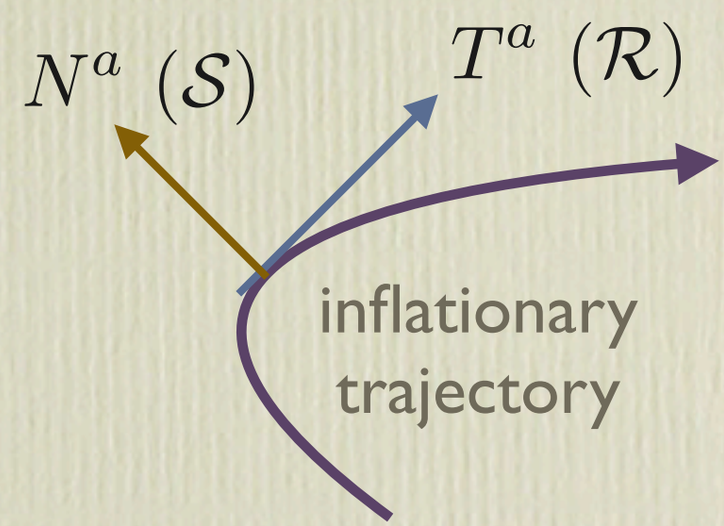
Adiabatic and entropy perturbations are given by

[Langlois, Renaux-Petel, '08]

$$\mathcal{R}_k'' + 2\frac{z'}{z}\mathcal{R}_k' + k^2\mathcal{R}_k = 2aH\omega\,\mathcal{S}_k' + \left(2aH\omega\,\frac{z'}{z} + 2\frac{(zaH\omega)'}{z}\right)\mathcal{S}_k$$

$$\mathcal{S}_k'' + 2\frac{z'}{z}\mathcal{S}_k' + \left(k^2 + \frac{z''}{z} - \frac{\alpha''}{\alpha} + a^2\,M_s^2\right)\mathcal{S}_k = -2aH\omega\,\mathcal{R}_k'$$

$$\frac{M_s^2}{H^2} = \frac{V_{NN}}{H^2} + \underline{M_{\rm Pl}^2\,\epsilon\,\mathbb{R} - \omega^2} \quad , \qquad \frac{z'}{z} = aH\,(1+\epsilon-\delta)$$



DYNAMICS OF LINEAR PERTURBATIONS

- The dynamics of the linear perturbations and cosmological predictions depends on the hierarchies of the adiabatic and entropy modes' masses relative to each other, the Hubble parameter, the turning rate ω, the curvature of the scalar manifold $\mathbb{R}$

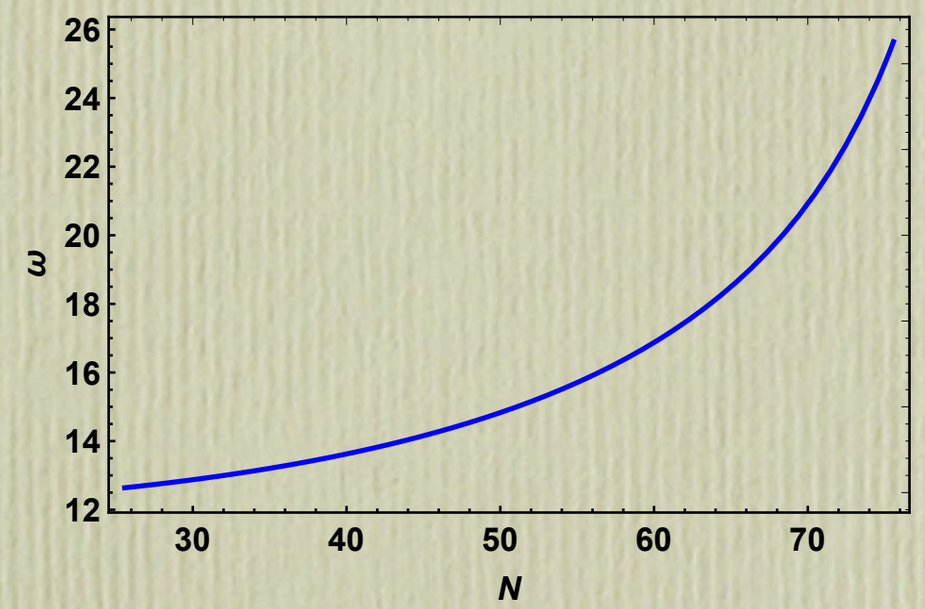
[Sasaki, Stewart, '96; Gordon, Wands, Bassett, Maartens, '00;
Groot Nibbelink, van Tent,'01; Langlois, Renaux-Petel, '08]
[Achúcarro, Gong, Hardeman, Palma, Patil, '10;
Achúcarro, Atal, Cespedes, Gong, Palma, Patil, '12;
Cespedes, Atal, Palma, '12 ...;
[Renaux-Petel, Turzynski, '15; Garcia-Saenz, Renaux-Petel, '18...]

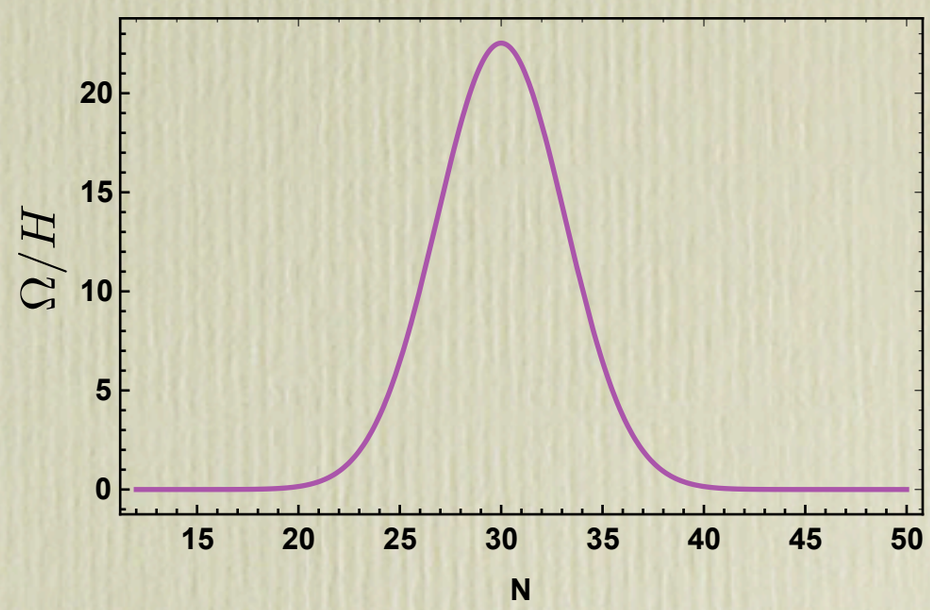
TRANSIENT LARGE TURNS: PBH AND GW

(Anguelova's talk)

- Transient strongly non-geodesic trajectories interesting phenomenology: PBHs, GWs

[Anguelova, Antal, Chen, Barausse, Braglia, Domenech, Finelli, Fumagalli, Hazra, Palma, Renaux-Petel, Riquelme, Ronayne, Scheihing, Sypsas, Slosar, Smoot, Sriramkumar, Starobinsky, Witkowski, Zenteno, ... '18-'22]

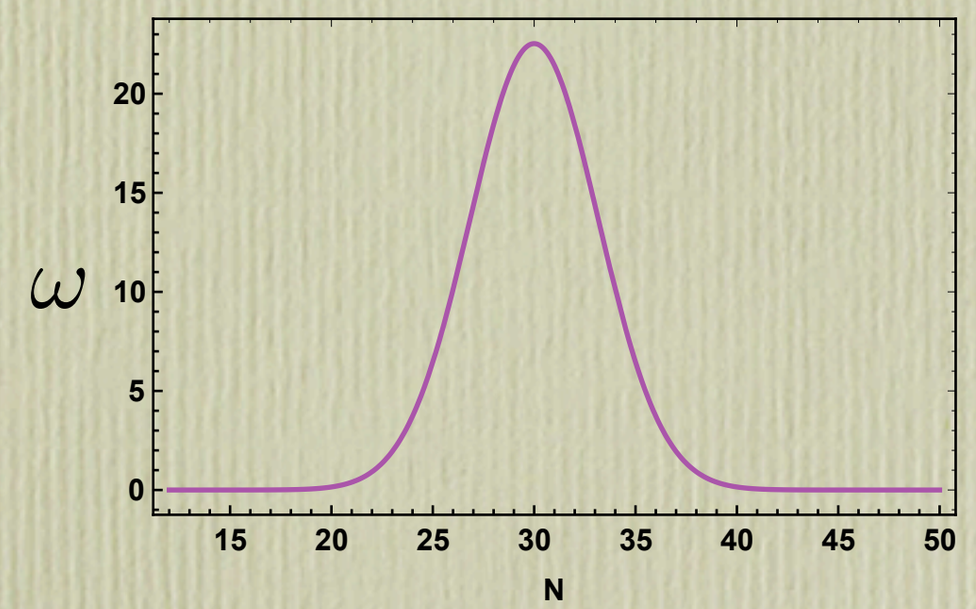




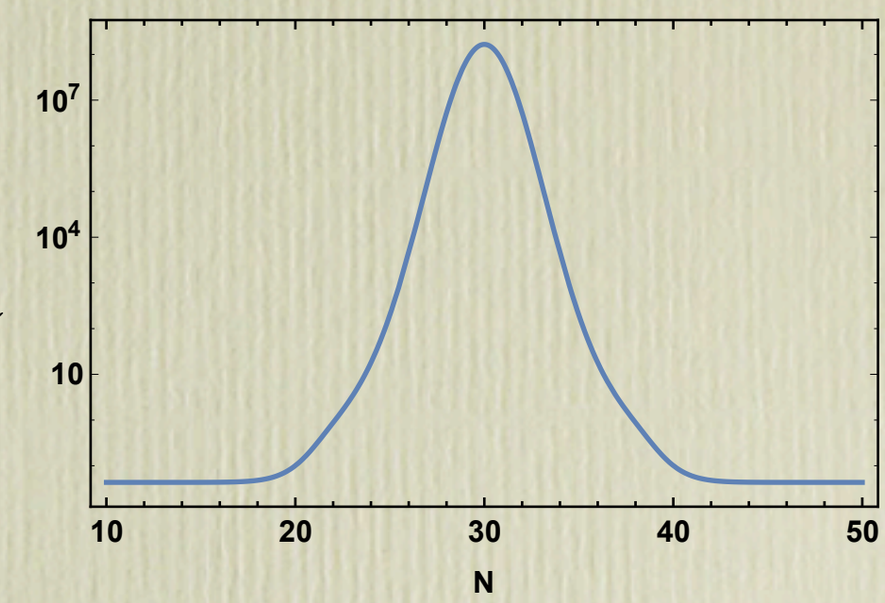
TRANSIENT LARGE TURNS: PBH AND GW

- Transient strongly non-geodesic trajectories interesting phenomenology: PBHs, GWs

[Anguelova, Antal, Chen, Barausse, Braglia, Domenech, Finelli, Fumagalli, Hazra, Palma, Renaux-Petel, Riquelme, Ronayne, Scheihing, Sypsas, Slosar, Smoot, Sriramkumar, Starobinsky, Witkowski, Zenteno, ... '18-'22]



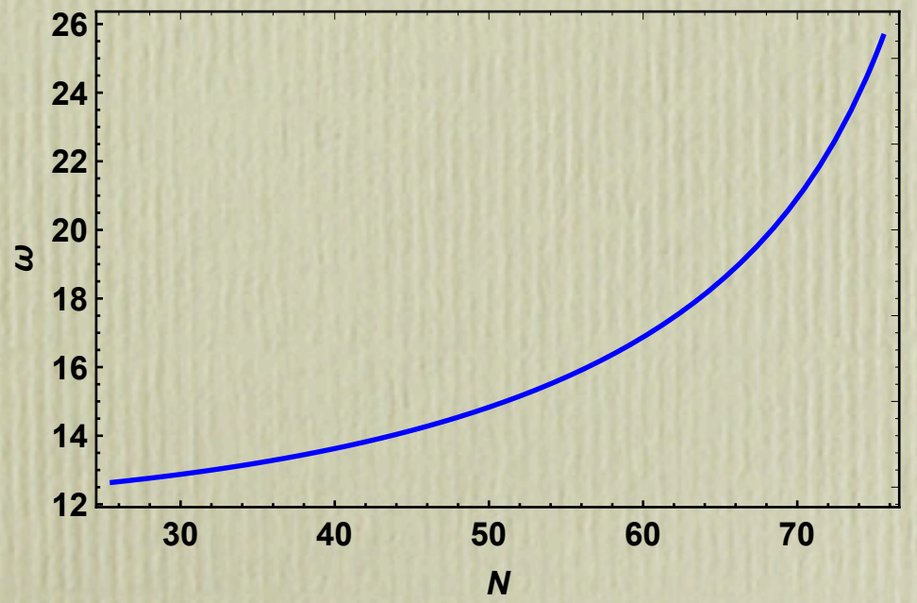
$\Rightarrow$ $\mathcal{P}_{\mathcal{R}}$

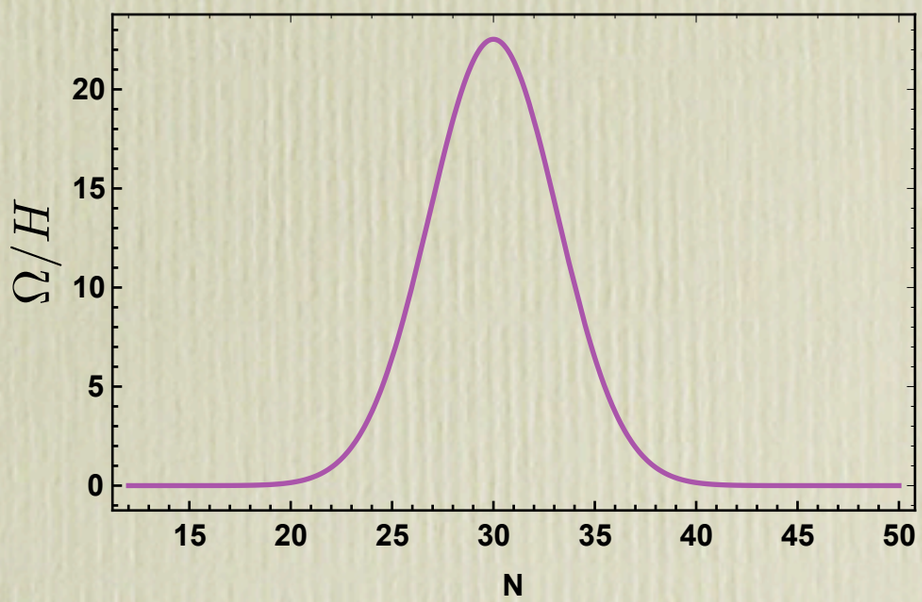


- Large tachyonic mass of entropic fluctuations results in their exponential growth, a transient instability which is transferred to the curvature perturbation

$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}_0\, e^{2x} \qquad (x \propto \omega)$$

LARGE & RAPID TURNS IN SUPERGRAVITY

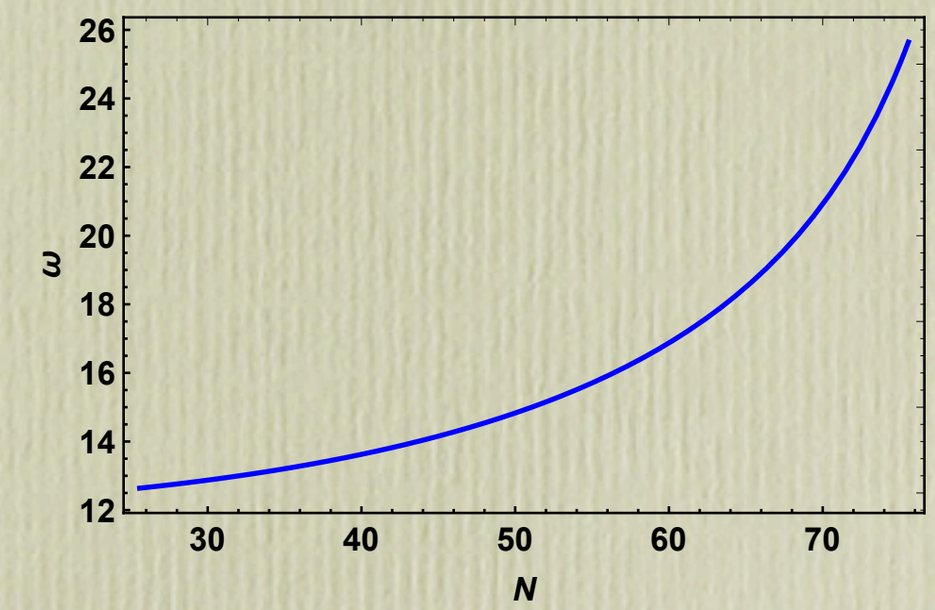




NON-GEODESIC ATTRACTORS IN SUPERGRGAVITY

- Large, slow turns in supergravity are hard to find $(\nu \ll 1)$

[Aragam, Chivoloni, Paban, Rosati, IZ, '21]



NON-GEODESIC ATTRACTORS IN SUPERGRGAVITY

- Large, slow turns in supergravity are hard to find $(\nu \ll 1)$

[Aragam, Chivoloni, Paban, Rosati, IZ, '21]

- We found a single example in the literature: EGNO

[Ellis, Garcia, Nanopoulos, Olive, '14]

$$K = -3\,\alpha \log\left[\Phi + \bar{\Phi} - c(\Phi + \bar{\Phi} - 1)^4\right] + \frac{S\bar{S}}{(\Phi + \bar{\Phi})^3}\,,$$

$$(\mathbb{R}(c, \alpha))$$

$$W = SF(\Phi)\,, \quad F(\Phi) = \sqrt{\frac{3}{4}}\,\frac{M}{a}\,(\Phi - a)\,,$$

$$[\mathbb{R} \sim (5 - 120)]$$

$$\left(\alpha = 1, \quad a = 1/2, \quad M = 10^{-3}, \quad c = 10^3\right) \quad \Rightarrow \quad \omega \sim 1.3$$

- Tune K, ☞ tuning (c, α) to increase ω :

$$\alpha = 10^{-3},\ c = 10^5 \quad \Rightarrow \quad \omega \sim 3$$

$$[\mathbb{R} > 0]$$

NON-GEODESIC ATTRACTORS IN SUPERGRGAVITY

[Aragam, Chivoloni, Paban, Rosati, IZ, '21]

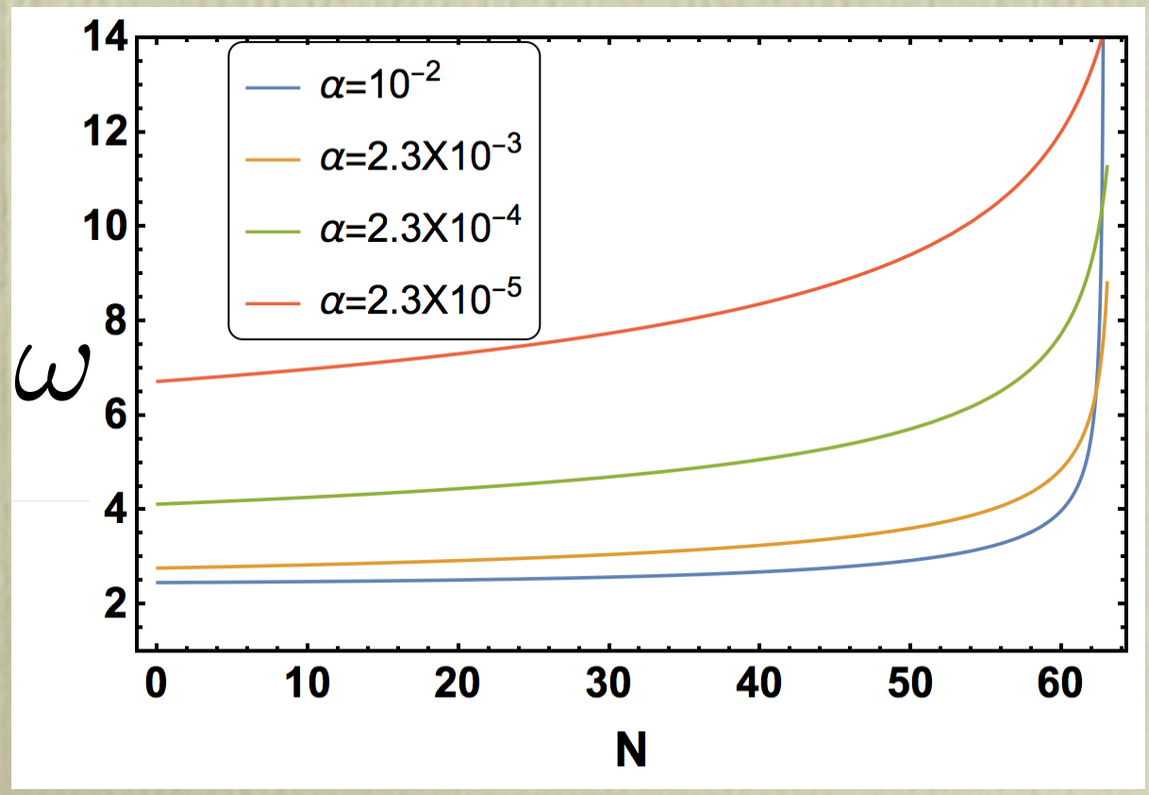
- No-scale inspired model

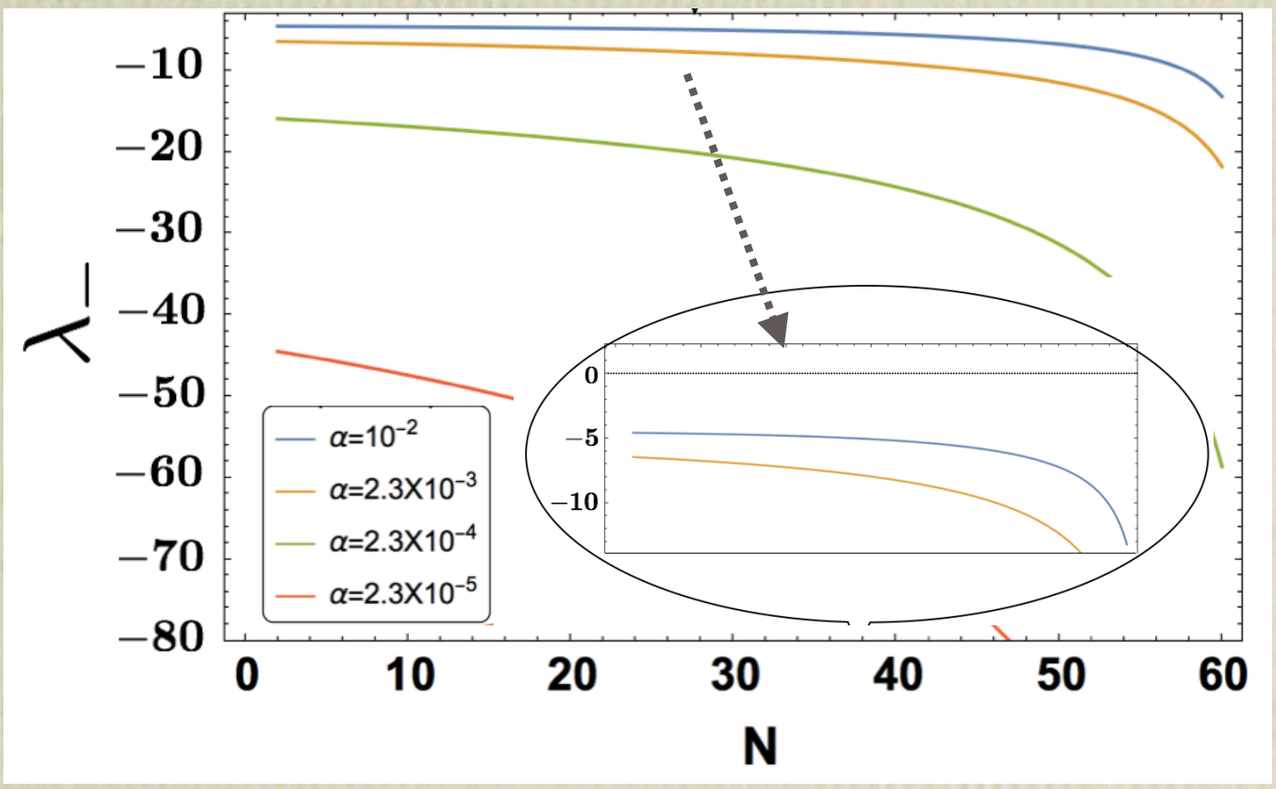
$$K = -3\,\alpha M_{\rm Pl}^2 \log[(\Phi+\bar\Phi)/M_{\rm Pl}] + S\bar S\,, \qquad F(\Phi) = p_0 + p_1\Phi\,. \qquad (W = SF(\Phi))$$

$$V = \frac{M_{\rm Pl}^{3\alpha}\,|F|^2}{(\Phi+\bar\Phi)^{3\alpha}}\,, \qquad \frac{\Omega}{H} \simeq \frac{2\sqrt{\epsilon_T}}{\sqrt{3\,\alpha}}\,. \qquad \left(\mathbb{R} = -\frac{4}{3\alpha}\right)$$

- By tuning α, ω can be made large

$(|\lambda_-| = \eta_V)$





SHARP TURNS WITH SMALL $\mathbb{R}$ $(\nu \gtrsim 1)$

[Bhattacharya, IZ, '22]

- A mechanism to generate transient large turns in supergravity without large (negative) curvature:

large turns arise due to transient violations of slow-roll

Natural set up:

axion monodromy inflation with subleading non-perturbative corrections

SHARP TURNS WITH SMALL $\mathbb{R}$ $(\nu \gtrsim 1)$

[Bhattacharya, IZ, '22]

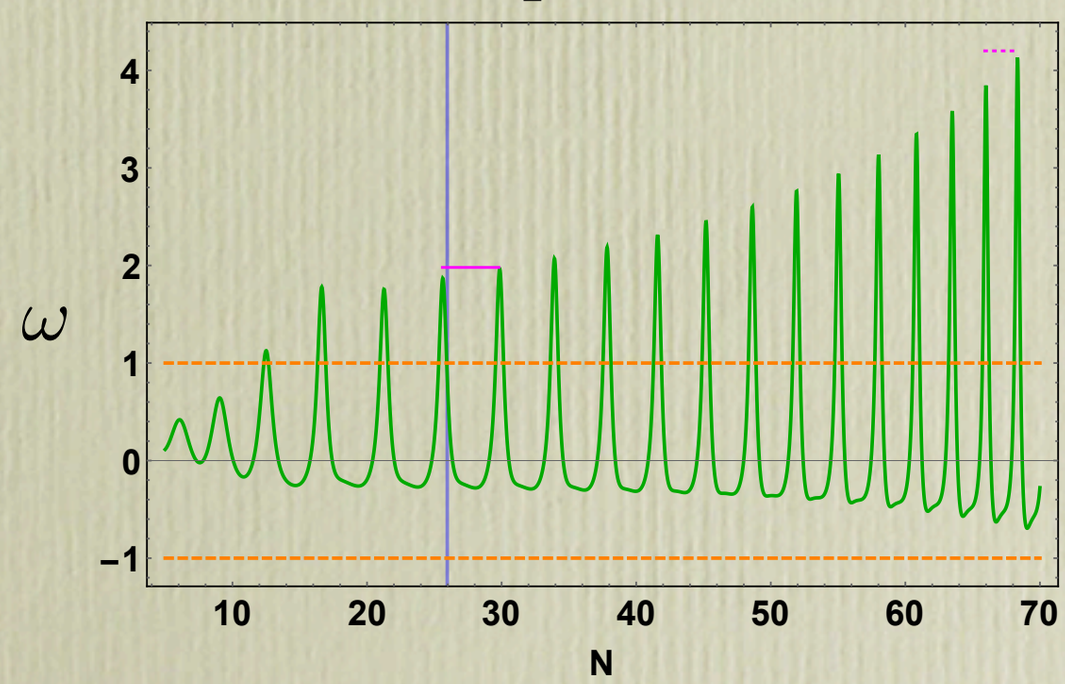
- A mechanism to generate transient large turns in supergravity without large (negative) curvatures arises through transient violations of slow-roll due to subleading corrections to axion

[Parameswaran, Tasinato, IZ, '16;
Cabo-Bizet, Loaiza-Brito, IZ, '16;
Özsoy, Parameswaran, Tasinato, IZ, '18]

$$K = -\log[\Phi + \bar{\Phi} - S\bar{S}]\,, \qquad (\mathbb{R} = -4)$$

$$W = S(M\Phi + ie^{-b\Phi}) \qquad (S^2 = 0)$$

$$V = \frac{M^2}{\beta}\left(\rho^2 + \theta^2 + \frac{2\lambda}{M}\,e^{-b\rho}\left[\theta\,\cos(b\,\theta) + \rho\,\sin(b\,\theta) + \frac{\lambda}{2M}\,e^{-b\rho}\right]\right)$$



sugra (multifield) axion monodromy

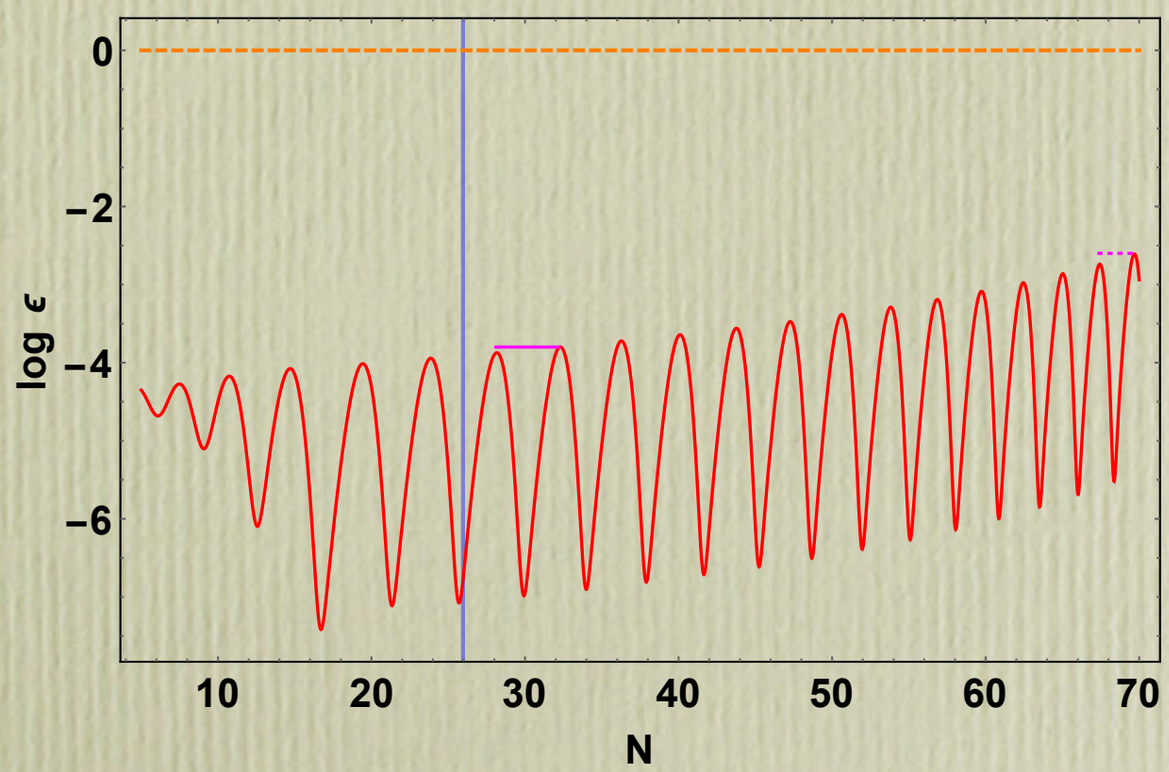
$$\lambda/M = 80\,, \quad b = 50$$

SHARP TURNS WITH SMALL $\mathbb{R}$

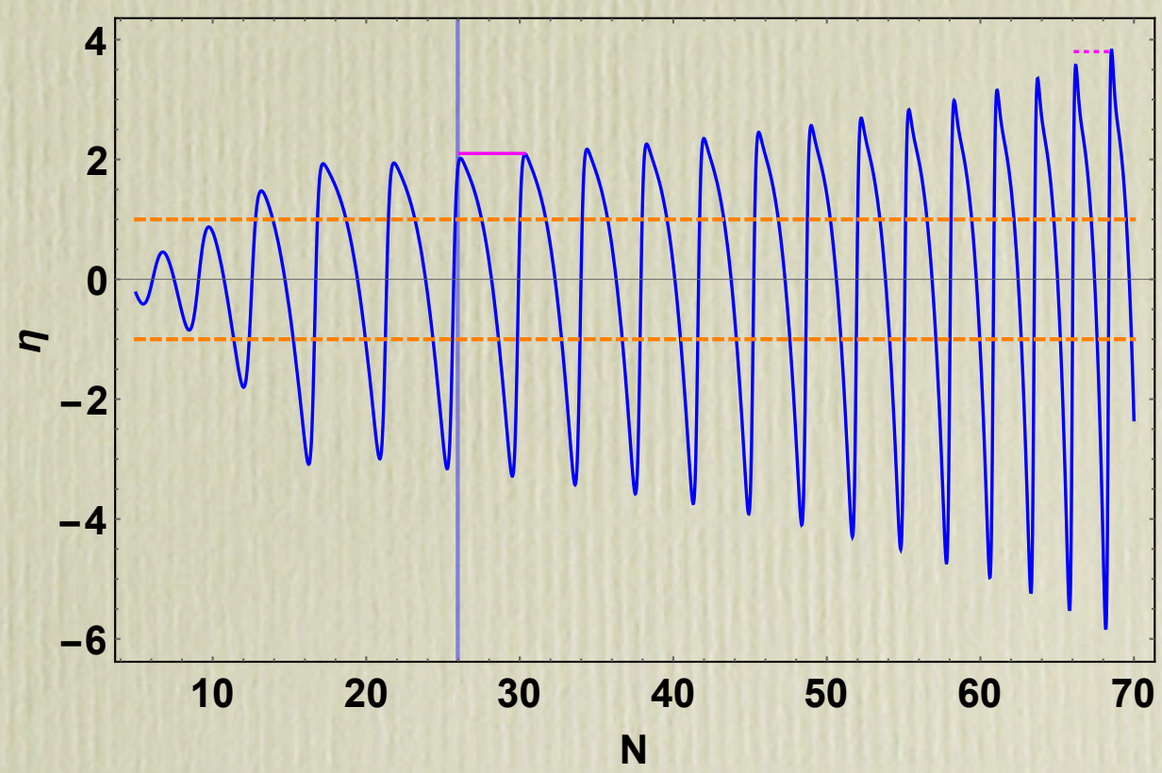
[Bhattacharya, IZ, '22]

- A mechanism to generate transient large turns in supergravity without large (negative) curvatures arises through transient violations of slow-roll due to subleading corrections to axion

$$\lambda/M = 80\,, \quad b = 50$$



$\epsilon \ll 1$



$\eta \gtrsim 1$

POWER SPECTRA

[Bhattacharya, IZ, '22]

- Large enhancement ($\sim \mathcal{O}(10^7)$) of adiabatic spectrum at small scales due to combined oscillatory effects

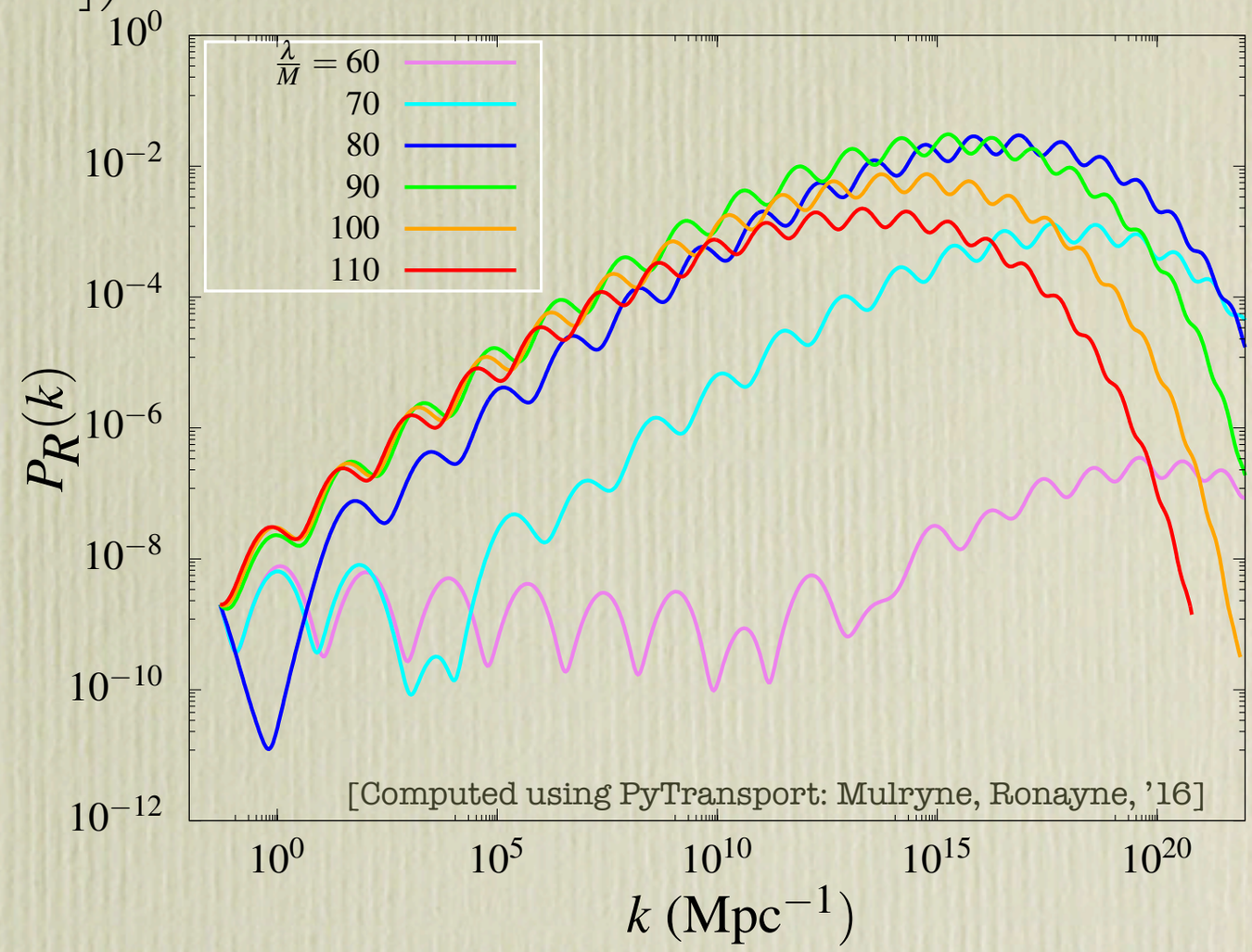
- Characteristic modulated enhanced spectrum

$$V = \frac{M^2}{\beta}\left(\rho^2 + \theta^2 + \frac{2\lambda}{M}\, e^{-b\rho}\left[\theta\,\cos(b\,\theta) + \rho\,\sin(b\,\theta) + \frac{\lambda}{2M}\, e^{-b\rho}\right]\right)$$

M	λ/M	b	$\rho_{\rm ini}$	$\theta_{\rm ini}$	$N_{\rm inf}$	r	$V_{\rm inf}^{1/4}$
2.52×10^{-6}	60	50	0.250	4.20	64.77	0.010	0.0029
2.73×10^{-6}	70	50	0.250	4.20	62.32	0.016	0.0030
2.15×10^{-6}	80	50	0.245	4.20	59.48	0.018	0.0027
6.41×10^{-7}	90	50	0.250	4.20	57.49	0.020	0.0015
1.10×10^{-7}	100	50	0.250	4.20	56.07	0.022	0.0006
1.25×10^{-8}	110	50	0.250	4.20	55.06	0.024	0.0002

$n_s = 0.9649 \pm 0.0042$ $\qquad (\beta = 1)$

- Subleading NP corrections change background evolution and cosmological predictions



LIGHT PBHS IN ABUNDANCE

[Bhattacharya, IZ, '22]

- Non-trivial PBHs mass spectrum with multiple peaks. (PBHs produced during radiation domination epoch) $(\gamma = 0.33)$

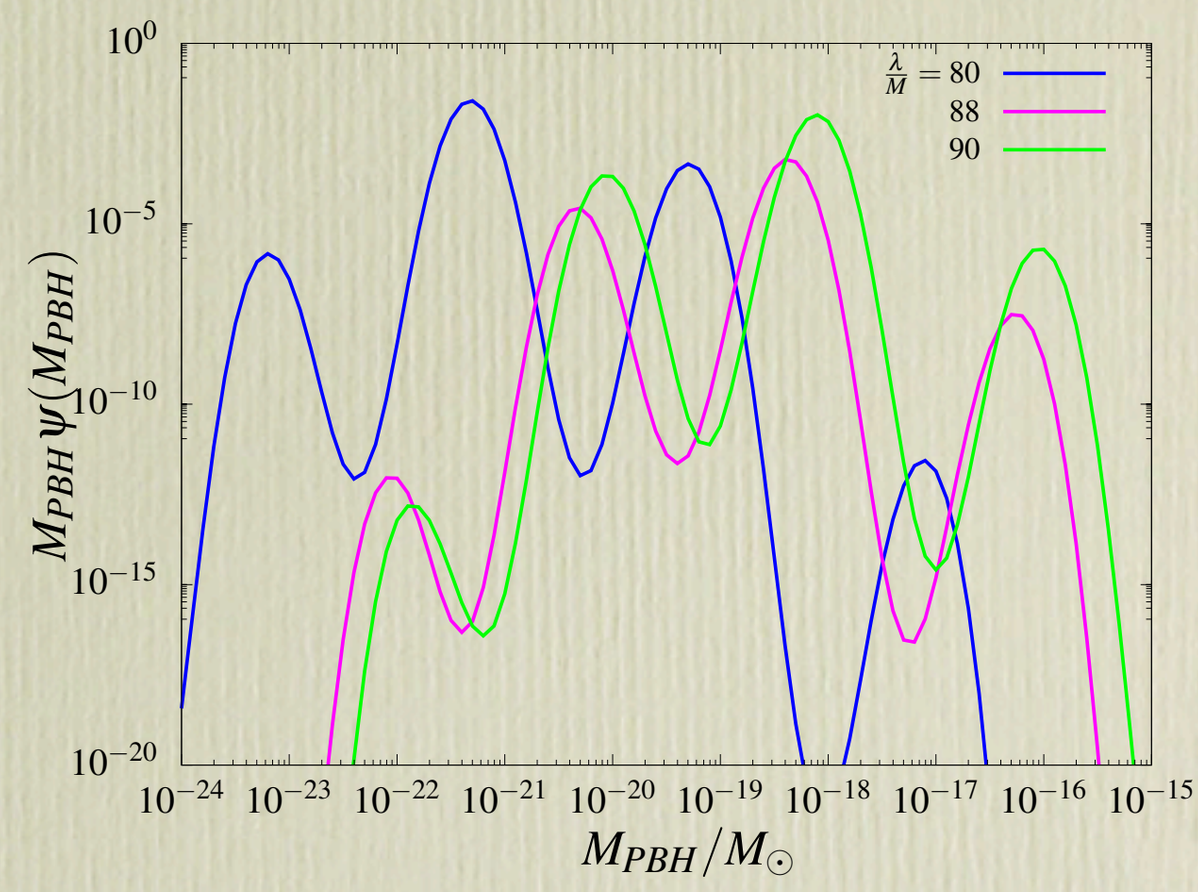
$$\psi(M_{\rm PBH}) = \frac{\gamma}{T_{\rm eq}} \left(\frac{g_s(T_1)}{g_s(T_{\rm eq})}\right)^{\frac{1}{3}} \left(\frac{\Omega_m h^2}{\Omega_c h^2}\right) \left(\frac{90 M_{\rm Pl}^2}{\pi^2 g_*(T_1)}\right)^{\frac{1}{4}} (4\pi\gamma M_{\rm Pl}^2)^{\frac{1}{2}} \frac{\beta_{\rm PBH}(M_{\rm PBH})}{M_{\rm PBH}^{\frac{3}{2}}}.$$

$(M_{PBH} = \gamma M_H \, ; \, M_{PBH} \sim k^{-2})$

- Multiple peaks can lead to abundance of PBHs in specific narrow mass ranges, while keeping the total abundance f_{PBH} small.

$$f_{\rm PBH} \equiv \frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}} = \int \psi(M_{\rm PBH}) dM_{\rm PBH}$$

$$f_{PBH} \sim 10^{-3} - 10^{-2}$$



LIGHT PBHS CONSTRAINTS

[Bhattacharya, IZ, '22]

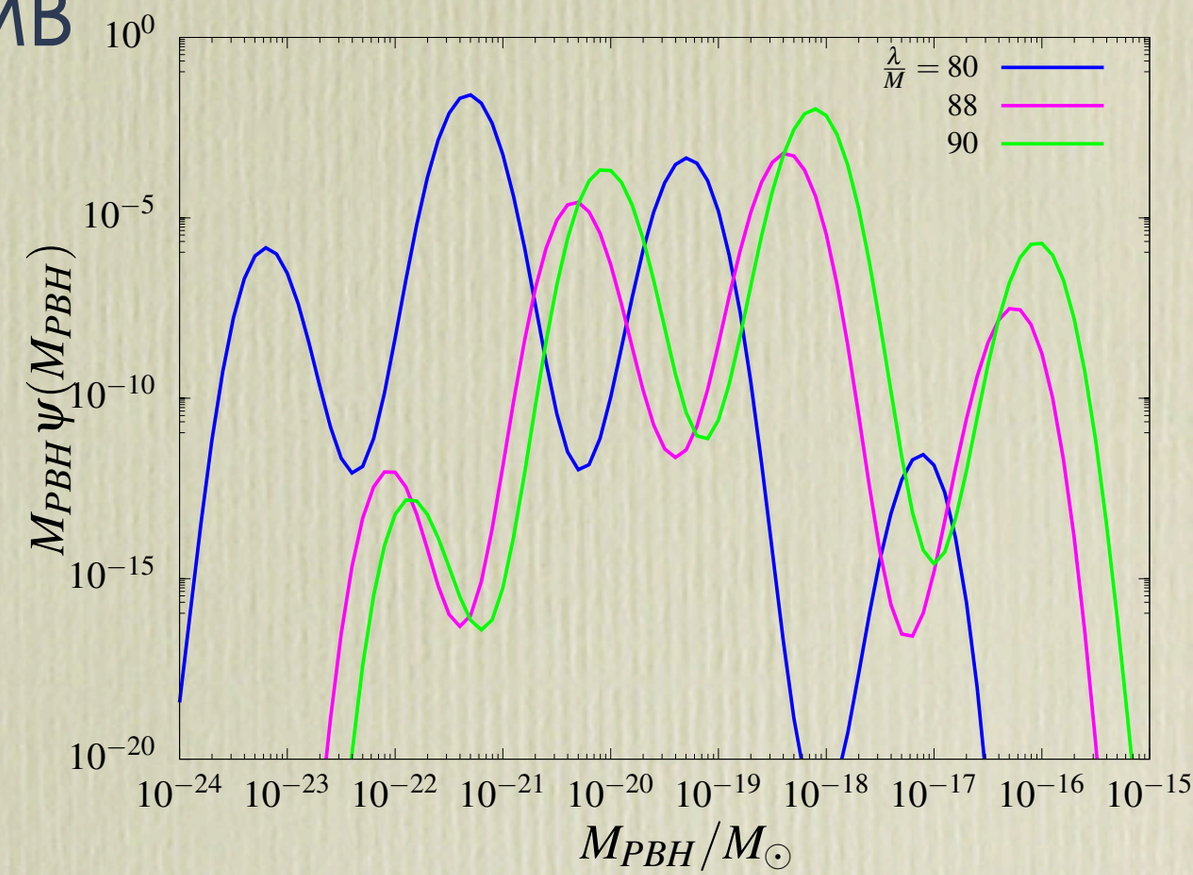
- While BBN constraints on f_{PBH} allow $f_{\rm PBH} \sim 10^{-4}$
- Stringent constraints from CMB and γ-ray observations give

$$f_{PBH} < 10^{-10}$$

for

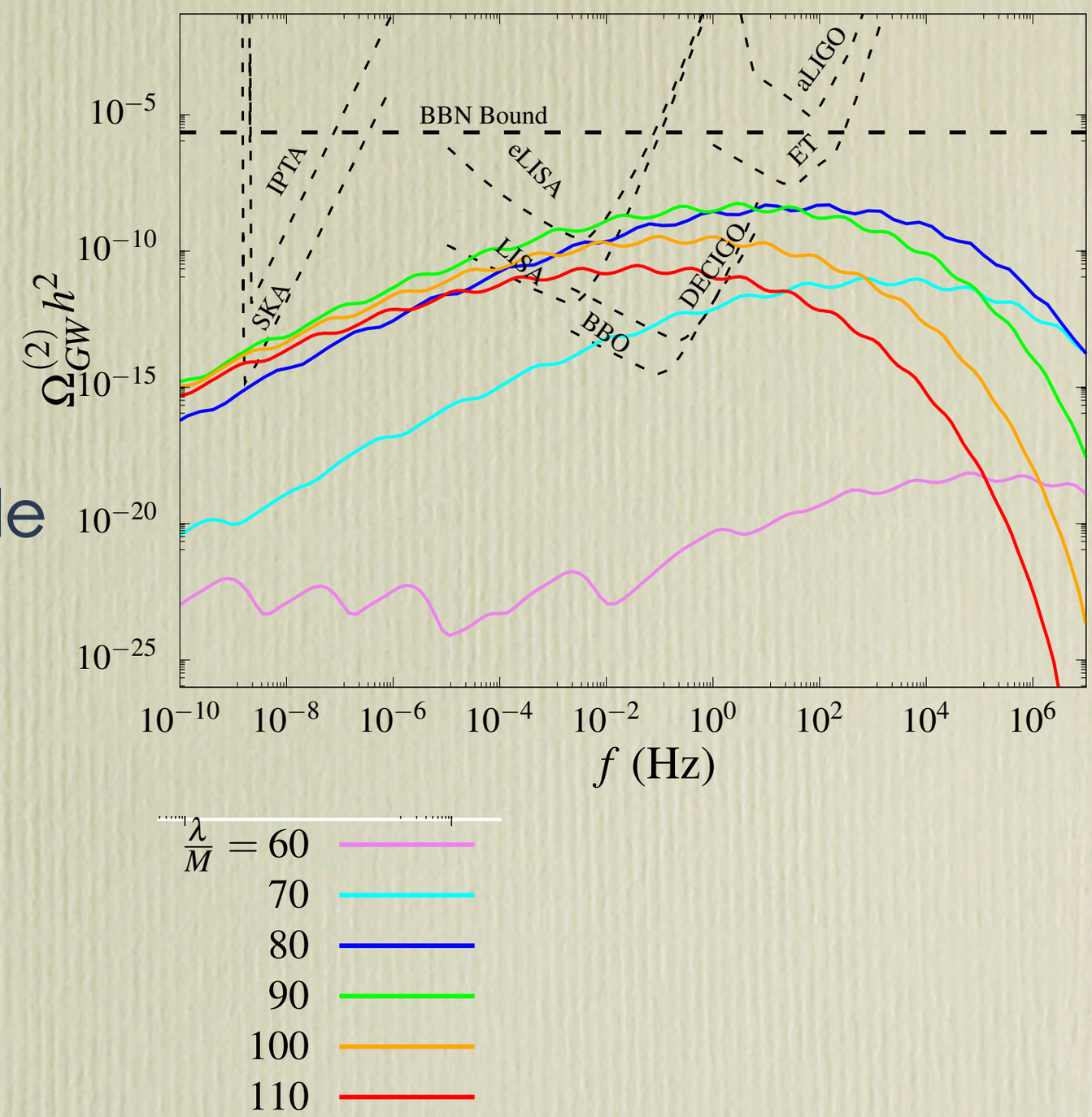
$$10^{-20} M_\odot \lesssim M_{PBH} \lesssim 10^{-17} M_\odot$$

(monochromatic $\psi(M_{PBH})$)



INDUCED GRAVITATIONAL WAVES

- Broad and large GW spectrum.
- Characteristic modulated shape
- Can be probed by multiple future surveys together



$$\Omega_{GW}^{(2)} \sim k^2 \tau^2 \mathcal{P}_h(k,\tau)$$

$$\mathcal{P}_h(k,\tau) \propto \mathcal{P}_{\delta\varphi}^2$$

SUMMARY

- Gravitational wave cosmology provides a unique opportunity to test fundamental theories of QG
- Primordial black holes and induced gravitational waves offer a direct window to the very early universe
- Curvature amplification mechanisms arise naturally in string inflation: axion monodromy, DBI
- Multifield inflation relaxes fine tunings encountered in single field amplification mechanisms
- Transient large turns induced from transient slow-roll violations offers a novel mechanism to generate strong non-geodesic trajectories in supergravity with R~1. Rich testable predictions: PBHs, IGWs